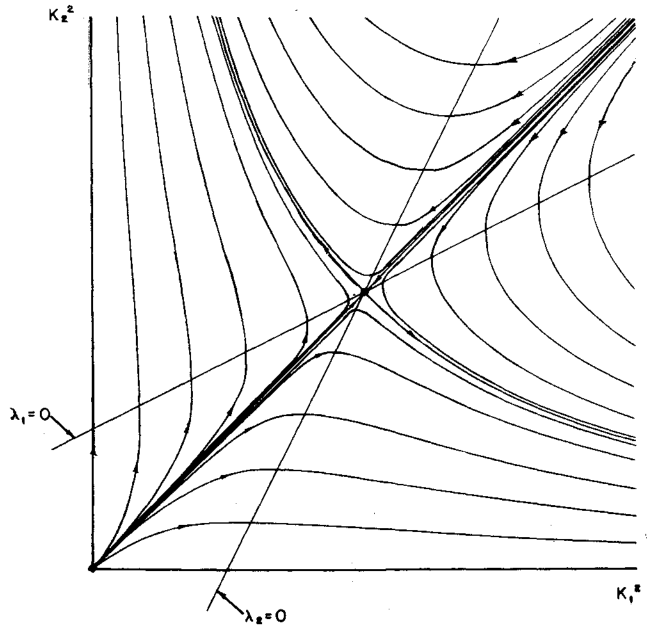

Fig. 1 Amplitude plane for unstable planar limit motion

According to the figure, exactly planar motions will go to a limit planar motion of maximum amplitude

$$K_1 + K_2 = 2|A + B|^{-1/2}$$

Large, nearly planar motion will decrease in amplitude, become elliptical and grow exponentially. Small motion and large, nearly circular motion immediately grow exponentially. Thus, the damping moment described by Eq. (1) would induce a planar limit motion in wind tunnel tests and would cause the angular motion to grow without bound in free flight. This example indicates the desirability of wind tunnel damping in pitch tests which allow the missile to move in a circular or elliptical motion.

References

[1] Murphy, C. H., "Circular pitching and yawing motion of cone configurations," *Proceedings of the Fourth AFBMD/STL Symposium* (Pergamon Press, New York, 1961), Vol. 2, pp. 328–336; also Ballistic Research Lab. Rept. 1071 (March 1959); also J. Aerospace Sci. **26,** 600–601 (1959).

[2] Murphy, C. H., "The prediction of nonlinear pitching and yawing motion of symmetric missiles," J. Aeronaut. Sci. **24,** 473–479 (1957); also Ballistic Research Lab. Rept. 995 (October 1956).

[3] Stoker, J. J., *Nonlinear Vibrations* (Interscience Publishers, Inc., New York, 1950).

[4] Murphy, C. H. and Hodes, B. A., "Planar limit motion of nonspinning symmetric missiles acted on by cubic aerodynamic moments," Ballistic Research Lab. Memo. Rept. 1358 (June 1961).

Results of Solid Rocket Motor Extinguishing Experiments

Richard W. Slocum Jr.*
American Space Industries, Inc., Palos Verdes, Calif.

An experiment was carried out to determine the feasibility of extinguishing a solid propellant rocket motor prior to normal burnout by injecting a suitable extinguishing chemical onto the burning surface of the grain. There, combustion would be stopped by either one or both of the following processes: 1) the temperature of the burning surface is lowered below the combustion temperature, and 2) the extinguishant acts as a chemical inhibitor of the combustion reactions.

Regarding extinguishants, a number of suggestions have been made, ranging all the way from water to more sophisticated chemicals that have been found effective in extinguishing more conventional types of chemical fires; however, few, if any, of these extinguishants have been tried in an operating motor. Specific mention may be made of certain halogenated hydrocarbons including some of the Freons,† e.g., FE 1301—bromotrifluoromethane ($CBrF_3$) and 114 B 2—dibromotetrafluoroethane ($CBrF_2$-$CBrF_2$), and bromochloromethane (CH_2BrCl). These chemicals are known to be excellent extinguishants of chemical fires.[1] It should be pointed out, however, that it is just as important for a "common use" extinguishant to have low toxicity as it is for it to have high extinguishing effectiveness. Thus there almost certainly are other compounds with greater extinguishing effectiveness than these, but which have not been classed as extinguishants because of their toxicity.

In the present case, bromochloromethane was selected as the extinguishant to be tried in an operating motor. This compound has a specific gravity of 1.95 and a boiling point of 152.6°F. It will be noted that this compound has one bromine atom per molecule. On the basis of data relating to flame speed in methane flames,[2] suggestions have been made that compounds with more than one bromine atom per molecule, or a greater proportion of bromine atoms, would be more effective as extinguishants. However, this is not supported by the data of Ref. 1 which show that on the average (of 7 compounds) those with 2 bromine atoms per molecule are not as effective as those with one, and none of those with 2 are as effective as bromochloromethane.

The motor used in the experiments had the following characteristics (when operating without injection of an extinguishant):

Propellant: 18% aluminum, 64% ammonium perchlorate, 18% PBAA and additives; flame temperature—5370°F; cylindrically shaped—4.75-in.-o.d. × 3.0-in.-i.d. × 7.38-in. long; flow rate—1.38 lb/sec

Chamber pressure: 430 psi

Thrust: 320 lb

Burning time: 3.7 sec

Injection was through the head end. To be sure the injectant reached the entire burning surface, a tube concentric with the longitudinal axis of the motor was used as injector. The injector extended the entire length of the grain and had an i.d. of 0.364 in. and o.d. of 0.540 in. 120-$\frac{1}{32}$-in. holes were drilled in the tube wall and spaced as follows: 30 circumferential rings of holes, each ring containing 4 holes (equally spaced around the circumference) and being perpendicular to the longitudinal axis of the injector; the relative phase of adjacent rings was 45°. Injection was initiated on command soon after motor ignition so that in use the injector was not damaged and could be reused.

Tests were carried out with two types of injector feed systems resulting in two different average flow rates of injectant *during motor operation.* These flow rates were 2.25 lb/sec and 6.15 lb/sec (±5%). In both cases, injectant tank pressure was 2000 psi (±10%). Calculations yield the following data: 1) average propellant velocity off the burning surface is 14.7 fps, 2) average injectant velocity at the surface of the injector is 26 fps at low flow and 70 fps at high.

When injection at the lower flow rate was initiated during motor operation and *continued until burnout,* the characteristic yellow-white exhaust plume remained basically yellow-white but trailed black "smoke"; this smoke is due to the

Received February 25, 1963.

* Physicist; presently Section Head, Propulsion Department, Systems Research and Planning Division, Aerospace Corporation, El Segundo, Calif.

† Dupont's trademark for its fluorocarbon compounds.

decomposition of the injectant. Injection was initiated one second after motor ignition and the motor burned for a total of 4.1 (±0.1) sec. Upon opening the motor case, it was found that all the propellant had burned except for some random patches of thickness no greater than about $\frac{1}{32}$ in. In tests using the high flow rate, upon commencing injection, the yellow-white plume immediately changed to a solid black plume of roughly the same dimensions as the plume before injection and stayed this way until burnout. Injection was initiated 1.5 sec after ignition and the motor burned for a total of 4.2 (±0.05) sec. The propellant residues were similar to those found in the low flow rate tests. Note that in both tests burning was not extinguished, but the total burning time was increased somewhat.

The foregoing tests indicate that, if solid motor thrust termination is to be achieved by injecting an extinguishant onto the burning surface, a concerted development program is needed. This should involve a coordinated program of research on extinguishants and on propellant additives which would serve to increase the effectiveness of specific extinguishants that would be used.

References

[1] "Freon," Dupont Tech. Bull. no. B-4B (1962).

[2] Levy, A., et al., "Inhibition of lean methane flames," *Eighth Symposium on Combustion* (Williams and Wilkins, Baltimore, Md., 1962), p. 524.

Matrix Derivation of a Short-Term Linear Rendezvous Equation

Richard Leach*
ITT Intelcom Inc., Bailey's Cross Roads, Va.

To optimize any system, one needs its equations of motion. If these equations of motion can be represented as a "linear plant," then optimization is relatively easy. By a "linear plant," one means that the equations of motion are in the form $\dot{x} = Fx + Gu$, where F and G are matrices that can be functions of time, x is the state vector, and u is the control vector.

Although the rendezvous problem can be linearized in rectangular coordinates, the problem is linearized better in a rotating coordinate system. For rendezvous with an object in a circular orbit and a rotating coordinate system, F is a constant matrix. For noncircular orbital rendezvous, the time-varying parts of F can be viewed as perturbations.

To obtain the equations of motion in a rotating coordinate system, first consider the matrix equation

$$x_r = Ax_s \tag{1}$$

where A is an orthogonal transformation matrix, x_s is a vector that represents a position in inertial space, and x_r is a vector that represents the same position in a rotated coordinate system. Now let A be a function of time, so that x_r is a function of time which represents a position x_s in a rotating coordinate system.

The matrix A has an inverse so that

$$x_s = A^{-1}(t)x_r \tag{2}$$

Differentiate twice with respect to time:

$$\dot{x}_s = \dot{A}^{-1}x_r + A^{-1}\dot{x}_r \tag{3}$$

$$\ddot{x}_s = \ddot{A}^{-1}x_r + 2\dot{A}^{-1}\dot{x}_r + A^{-1}\ddot{x}_r \tag{4}$$

Received by ARS September 27, 1962.

* Member of the Technical Staff.

By Newton's laws,

$$\ddot{x}_s = f_s/m \tag{5}$$

where f_s is the force acting on a body of mass m. To write the force in the rotating coordinate system, f_r, one must matrix-multiply Eq. (5) by A:

$$f_r/m = A(f_s/m) = A\ddot{x}_s \tag{6}$$

Using Eq. (4),

$$\ddot{x}_r = (f_r/m) - 2A\dot{A}^{-1}\dot{x}_r - A\ddot{A}^{-1}x_r \tag{7}$$

The last two terms in Eq. (7) can be considered as "inertial" forces. The $-2A\dot{A}^{-1}\dot{x}_r$ is the generalized coriolis force, and the $-A\ddot{A}^{-1}x_r$ is the generalized centrifugal force.

For a frame rotating at a constant speed, these generalized inertial forces are identical to the classical ones. Let the frame rotate at a constant speed about the z axis of the inertial frame; then

$$A = \begin{pmatrix} \cos\omega t & \sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8}$$

Let ω be a vector whose magnitude defines the rotation rate and whose direction defines the axis of rotation. Then

$$-2A\ddot{A}^{-1}\dot{x}_r = -2\begin{pmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\dot{x}_r = -2(\omega \times \dot{x}_r) \tag{9}$$

which is the usual way of representing the coriolis force. Now

$$-A\ddot{A}^{-1}x_r = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}x_r = -\omega \times (\omega \times x_r) \tag{10}$$

where the cross product is defined in terms matrix multiplication. This is the usual representation of centrifugal force.

Having obtained the generalized inertial force, the usual orbital equation can be obtained from Eq. (7). Fixing the axis of rotation in the z direction and replacing ωt by θ, Eq. (7) becomes

$$\ddot{x}_r + 2\begin{pmatrix} 0 & -\dot{\theta} & 0 \\ \dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\dot{x}_r + \begin{pmatrix} -\dot{\theta}^2 & -\ddot{\theta} & 0 \\ \ddot{\theta} & -\dot{\theta}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}x_r = \frac{f_r}{m} \tag{11}$$

Examine the equations of the individual components. Let

$$X_r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

If one requires that the inertial values of z and $\dot{z}$ be zero and that the z component of f_r for $z = 0$ be zero, then $z(t) = 0$. Let $y = r$, require $x(t) = 0$, and let f_r be a central force:

$$_r = \begin{pmatrix} 0 \\ f_r \\ 0 \end{pmatrix}$$

Then the equation for the x component is

$$-2\dot{\theta}\dot{r} - \ddot{\theta}r = 0 \tag{12}$$

The equation for the r component or y is

$$\ddot{r} - \dot{\theta}^2 r = f_r/m \tag{13}$$

Equations (12) and (13) are the usual orbit equations.

To find the linear rendezvous equations easily, one uses the usual linearization technique. Define

$$x_\Delta = X_r - x_k \tag{14}$$

where X_r is a general solution in the rotating coordinates of Eq. (11), and x_k is a specific known solution. One expands